Marking Scheme

Compiled by Joe

Section A(1)

1. $\frac{(x^5y)^{-4}}{x^{14}y^{20}} = \frac{x^{-20}y^{-4}}{x^{14}y^{20}} = x^{-20-14}y^{-4-20} = x^{-34}y^{-24} = \frac{1}{x^{34}y^{24}}$ 1M + 1M + 1A

2. (a) $4x^2 + 28xy + 49y^2 = (2x + 7y)^2$ 1A

   (b) $4x^2 + 28xy + 49y^2 - 8x - 28y$

   $= (2x + 7y)^2 - 4(2x + 7y)$ 1M

   $= (2x + 7y)(2x + 7y - 4)$ 1A

3. $\frac{0 \times 3 + 3 \times 6 + 6 \times 7 + 10 \times p}{3 + 6 + 7 + p} = 5$ 1M

   $\frac{60 + 10p}{16 + p} = 5$

   $60 + 10p = 80 + 5p$ 1M

   $5p = 20$

   $p = 4$ 1A

4. (a) For $n = 6$,

   $\sin (5 \times 6)° = \sin 30° = 0.5$ 1A

   (b) $\frac{\sqrt{2}}{2}$, $\frac{\sqrt{3}}{2}$ 2A

5. (a) $3(h-4k) = 6h-5$

$3h-12k = 6h-5$ 1M

$-12k = 3h-5$ 1M

$k = \frac{3h-5}{-12}$

$k = \frac{5-3h}{12}$ 1A

(b) $k = \frac{5-3h}{12}$

$k' = \frac{5-3(h-4)}{12} = \frac{5-3h+12}{12} = \frac{5-3h}{12} + \frac{12}{12} = k+1$

$\therefore$ k is increased by 1. 1A

k 增加 1。

6. (a) $\frac{7x-2}{3} < 5(x+4)$ and $9-3x \geq 0$

$7x-2 < 15(x+4)$ and $-3x \geq -9$

$7x-2 < 15x+60$ and $x \leq 3$

$-8x < 62$ and $x \leq 3$

$x > -\frac{31}{4}$ and $x \leq 3$ 1M + 1M

$\therefore$ $-\frac{31}{4} < x \leq 3$ 1A

(b) 7 1A

7. (a) Height of Susan 美玲的高度

$=\dfrac{165}{(1+10\%)}$ 1M

$=\dfrac{165}{1.1}$

$=150$ cm 1A

(b) Height of Amy 佩詩的高度

$=150(1-10\%)$

$=135$ cm 1A

The required percentage 所求百分率

$=\dfrac{165-135}{165}\times 100\%$

$=18\dfrac{2}{11}\%$ (or 18.2%) 1A

8. (a) L is the angle bisector of $\angle AOB$. 1A

L 為 $\angle AOB$ 的角平分線。

(b) Let $M(r, \theta)$ be the polar coordinates of the point of intersection of L and AB.

設 L 與 AB 的交點的極坐標為 $M(r, \theta)$。

$\angle AOB = 110° - 50° = 60°$ 1A

$\angle MOB = 30°$

$\cos 30° = \dfrac{r}{12}$ 1M

$r = 6\sqrt{3}$ 1A

$\theta = 50° + 30° = 80°$ 1A

$\therefore \quad M(6\sqrt{3}, 80°)$

9. Let x and y be the number of matches that team F wins and draws respectively.

設球隊 F 勝出的場數與賽和的場數分別為 x 及 y。

$$\begin{cases} \dfrac{x}{y} = \dfrac{3}{1} & \text{......(1)} \\ 3x + y = 50 & \text{......(2)} \end{cases}$$ 1A + 1A

From (1): $x = 3y$ ……(3)

Put (3) into (2),

$3(3y) + y = 50$ 1A

$y = 5$

Put $y = 5$ into (3),

$x = 3(5) = 15$ 1A

$\therefore$ $x = 15$, $y = 5$

$\therefore$ Number of matches that team F loses 球隊 F 輸掉的場數

$= 27 - 15 - 5 = 7$ 1M + 1A

SECTION A(2)

10. (a) $f(x)=(x+m)(x+n)(x-1)+5$

$f(0)=(0+m)(0+n)(0-1)+5$ 1M

$0=-mn+5$

$mn=5$ 1A

$\therefore\quad m=-5,\ n=-1$ 1A

(b) $f(x)=(x-5)(x-1)(x-1)+5=x^3-7x^2+11x$ 1M

$f(x)-g(x)=x^3-7x^2+11x-(x^3-9x^2+10x+k)=2x^2+x-k$ 1M

$f(x)-g(x)=0$

$2x^2+x-k=0$

$\Delta\geq 0$ 1M

$(1)^2-4(2)(-k)\geq 0$

$1+8k\geq 0$

$k\geq -\dfrac{1}{8}$ 1A

11. (a) $C = k_1 + \frac{k_2 A}{n}$ where k_1 and k_2 are non-zero constants 其中 k_1 及 k_2 為非零常數 1A

When n = 100 and A = 400, C = 1.8

$$1.8 = k_1 + \frac{k_2(400)}{100}$$

$$1.8 = k_1 + 4k_2 \qquad \text{.......(1)}$$

When n = 200 and A = 500, C = 1.5

$$1.5 = k_1 + \frac{k_2(500)}{200}$$

$$1.5 = k_1 + 2.5k_2 \qquad \text{.......(2)}$$ 1M

By solving (1) & (2), k_1 = 1, k_2 = 0.2 1A

$$\therefore \quad C = 1 + \frac{0.2A}{n}$$

When n = 300 and A = 360,

$$C = 1 + \frac{0.2(360)}{(300)} = 1.24$$

The total cost 總成本

$= 1.24 \times 300$

$= \$372$ 1A

(b) Agree 同意 1A

When n tends to infinity, $\frac{0.2A}{n}$ tends to 0, 1M

當 n 趨向無限大時，$\frac{0.2A}{n}$ 趨向 0，

$$\therefore \quad C = 1 + \frac{0.2A}{n} = 1 + 0 = 1$$ 1M

12. (a) Maximum absolute error 最大絕對誤差 = 5 mL 1A

(b) Upper limit 上限 = 800 + 5 = 805 mL

Lower limit 下限 = 800 – 5 = 795 mL 1M

∴ $795 \le x < 805$ 1A

(c) (i) Upper limit of 48 packs of orange juice

48 包橙汁的總體積上限

$= 805 \times 48$ 1M

$= 38640$ mL

$= 38.640$ L 1M

< 38.7 L

∴ It is impossible 不可能 1A

(ii) Lower limit of a glass of orange juice

一杯橙汁的下限

$= 250 - 2.5$

$= 247.5$ mL

Number of glass of orange juice

橙汁的杯數

$= \dfrac{38640}{247.5}$ 1M

≈ 156.12

< 157

∴ It is impossible 不可能 1A

13. (a) (i) Outer total surface area 外側的總表面面積

$= 20 \times 40 \times 2 + 20 \times 30 \times 2 + 40 \times 30$ 1M

$= 4000\text{ cm}^3$ 1A

(ii) Capacity 容量

$= 20 \times 40 \times 30$ 1M

$= 24000\text{ cm}^3$ 1A

(b) (i) $\tan\theta = \frac{20}{40} = \frac{1}{2}$ 1M

$\theta = 26.6°$ 1A

(ii) $\angle ADA' = \theta$ 1M

$\tan\angle ADA' = \frac{AA'}{AD}$

$\tan\theta = \frac{AA'}{40}$ 1M

$\frac{1}{4} = \frac{AA'}{40}$

$AA' = 10$ 1A

Area of $A'DEF$ = area of $ADEF$ – area of $\Delta ADA'$

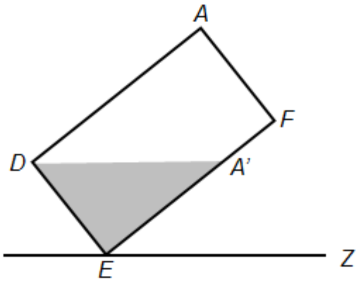
$A'DEF$ 的面積 = $ADEF$ 的面積 – $\Delta ADA'$ 的面積

$X = 20 \times 40 - \frac{40 \times 10}{2} = 600$ 1M + 1A

(iii) $X = 200$

$\frac{DE \times EA'}{2} = 200$

$EA' = 20$ 1M



$\therefore \tan\theta = \frac{DE}{EA'} = \frac{20}{20} = 1$

$\theta = 45°$ 1A

SECTION B

14. (a) For Y: When $N = 2.5$,

$$2.5 = \log_{100}\frac{E}{10}$$ 1M

$$100^{2.5} = \frac{E}{10}$$

$$10^5 = \frac{E}{10}$$

$$10^6 = E$$ 1A

For X: When $E = 10^6$, $M = 8$

$$8 = \log 10^6 + k$$ 1M

$$8 = 6 + k$$

$$k = 2$$ 1A

(b) For X: When $M = a$

$$a = \log E + 2$$

$$a - 2 = \log E$$

$$E = 10^{a-2}$$ 1M

For Y: When $E = 10^{a-2}$,

$$N = \log_{100}\frac{10^{a-2}}{10}$$ 1M

$$N = \log_{100}10^{a-3}$$

$$N = (a-3)\log_{100}10$$ 1M

$$N = (a-3)(\frac{1}{2})$$

$$N = \frac{a-3}{2}$$ 1A

15. (a) $\frac{1}{2}yz\sin\theta = 12$ 1M

$yz = \frac{24}{\sin\theta}$ 1A

(b) $\sin\theta = \frac{6}{10} = 0.6$ ⇨ $yz = \frac{24}{0.6} = 40$ 1A

$x^2 = y^2 + z^2 - 2yz\cos\theta$ 1M

$= y^2 + z^2 - 2(40)(\frac{8}{10})$

$= y^2 + z^2 - 64$

$= y^2 - 2yz + z^2 + 2yz - 64$

$= (y-z)^2 + 2(40) - 64$

$= (y-z)^2 + 16$ 1M

∴The minimum value of x

x 的最小值

$= \sqrt{16}$

$= 4$ 1A

16. (a) (i) Γ and ℓ are parallel. 1A

Γ 及 ℓ 互相平行。

(ii) Slope of Γ = slope of L

Γ 的斜率 = L 的斜率

$$= \frac{-3-0}{0-6} = \frac{1}{2}$$ 1M

y-intercept of Γ

Γ 的 y-截距

$$= \frac{-3+(-7)}{2} = -5$$ 1M

∴ The required equation 所求方程

$$y = \frac{1}{2}x - 5$$ 1A

(b) (i) $G(10, 0)$

Put $G(10, 0)$ into $y = \frac{1}{2}x - 5$, 1M

代 $G(10, 0)$ 入 $y = \frac{1}{2}x - 5$，

LHS = y = 0

RHS = $\frac{1}{2}x - 5 = \frac{1}{2}(10) - 5 = 0$

∴ Yes 是。 1A

(ii) $GR = GS$ = radius 半徑 1M

Γ and ℓ are parallel so keep a fixed distance.

Γ 及 ℓ 互相平行，所以保持固定距離。

∴ Area of ΔPGR : area of ΔQGS = 1 : 1 1A

ΔPGR 的面積 : ΔQGS 的面積 = 1 : 1

17. (a) (i) Required probability 所求概率

$= \frac{1}{C_3^9} = \frac{1}{84}$ 1M + 1A

(ii) Required probability 所求概率

$= \frac{C_3^4}{C_3^9} = \frac{1}{21}$ 1M + 1A

(b) $p \cdot \frac{1}{21} + (1-p) \cdot \frac{1}{84} \le \frac{1}{70}$ 1M

$20p + 5 - 5p \le 6$

$15p \le 1$

$p \le \frac{1}{15}$ 1A

The largest value of proportion (p) 最大比例(p)

$= \frac{1}{15}$ 1A

(c) (i) Required probability 所求概率

$= \frac{1}{15} \cdot \frac{C_4^4}{C_4^9} = \frac{1}{1890}$ 1M + 1A

(ii) Required probability 所求概率

$= \frac{1}{15} \cdot \frac{C_3^4 \cdot C_1^5}{C_4^9} + \frac{14}{15} \cdot \frac{C_3^3 \cdot C_1^6}{C_4^9} = \frac{52}{945}$ 1M + 1A

(iii) Let q_1 = P(at most 1 smiley face on 1 card 在 1 張卡上刷中最多 1 個「笑臉」)

$= \frac{1}{15} \cdot (\frac{C_4^5}{C_4^9} + \frac{C_1^4 \cdot C_3^5}{C_4^9}) + \frac{14}{15} \cdot (\frac{C_4^6}{C_4^9} + \frac{C_1^3 \cdot C_3^6}{C_4^9}) = \frac{73}{126}$

Let q_2 = P(exactly 2 smiley faces on 1 card 在 1 張卡上確切地中兩個「笑臉」)

$= \frac{1}{15} \cdot \frac{C_2^4 \cdot C_2^5}{C_4^9} + \frac{14}{15} \cdot \frac{C_2^3 \cdot C_2^6}{C_4^9} = \frac{23}{63}$

∴ Required probability 所求概率

$= 1 - (q_1^2 + C_1^2 \cdot q_1 \cdot q_2)$ 1M

$= \frac{1277}{5292}$ 1A

1. A

$$(-7)^{300}\cdot\frac{1}{7^{100}}=(7)^{300}\cdot\frac{1}{7^{100}}=7^{300-100}=7^{200}$$

2. B

$$4-x^2+4xy-4y^2=4-(x^2-4xy+4y^2)=2^2-(x-2y)^2=(2+x-2y)(2-x+2y)$$

3. C

$$\frac{27\times30+22\times20}{50}=25\text{ page頁/min分鐘}$$

4. B

$$\text{Cost 成本}=\frac{700}{1+40\%}=\$500$$

$$\text{Profit 盈利}=700(1-15\%)-500=\$95$$

5. A

$p=-5$, $q=-1$, $r=2$.

$$\frac{p-q}{r}=\frac{-5-(-1)}{2}=-2$$

6. D

$hx(x+2)+x^2\equiv3kx(x+1)+4x$

Put $x=-1$,

$$h(-1)(-1+2)+(-1)^2=3k(-1)(-1+1)+4(-1)$$
$$h(-1)(1)+1=0-4$$
$$-h+1=-4$$
$$h=5$$

7. B

$AB = \sqrt{(-2-1)^2 + (0-5)^2} = \sqrt{34}$

For Choice A: $AC = \sqrt{(-2-1)^2 + (5-5)^2} = 3$

For Choice B: $AC = \sqrt{(4-1)^2 + (0-5)^2} = \sqrt{34}$

For Choice C: $AC = \sqrt{(0-1)^2 + (-4-5)^2} = \sqrt{82}$

For Choice D: $AC = \sqrt{(4-1)^2 + (1-5)^2} = 5$

8. D

$h + k = 2(360^o) - 70^o = 650^o$

9. B

Let x and y be the number of pens and rubbers bought respectively.

設 x 及 y 分別為購買了的原子筆及橡皮擦數量。

$$\begin{cases} 4.5x + 2.8y = 92.7 \text{ ....(1)} \\ x = 2y - 3 \text{ ....(2)} \end{cases}$$

Sub. (2) into (1),

$$\begin{aligned} 4.5(2y-3) + 2.8y &= 92.7 \\ 9y - 13.5 + 2.8y &= 92.7 \\ 11.8y &= 106.2 \\ y &= 9 \end{aligned}$$

Put $y = 9$ into (2), $x = 2(9) - 3 = 15$

∴ The total number of pens and rubbers bought

購買了的原子筆及橡皮擦的總數

= 9 + 15 = 24

10. B

∵ The graph opens downwards.

圖像開口向下。

∴ $a < 0$

The roots of the graphs are –3 and 8.

圖像的根為–3 及 8。

I. ✓

II. $y = -\frac{1}{2}x^2 + \frac{5}{2}x + 12 = -\frac{1}{2}(x^2 - 5x - 24) = -\frac{1}{2}(x+3)(x-8)$ ✓

III. $y = -15x - (72 - 3x^2) = -15x - 72 + 3x^2 = 3x^2 - 15x - 72$ ✗

11. B

$f(x) = 2x^2 + kx - 1$

$$f(2) = f\left(\frac{1}{2}\right)$$

$$2(2)^2 + k(2) - 1 = 2\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - 1$$

$$8 + 2k - 1 = \frac{1}{2} + \frac{1}{2}k - 1$$

$$k = -5$$

12. D

∵ It has a maximum point at (3, 6).

它於(3, 6)有極大點。

∴ $a < 0$, $h = 3$ and $k = 6$.

∴ The function is $f(x) = a(x-3)^2 + 6$ and a is negative.

函數為 $f(x) = a(x-3)^2 + 6$ 及 a 為負數。

13. C

Let $x = -2$ and $y = -1$

I. $\frac{2}{3x} = \frac{2}{3(-2)} = -\frac{1}{3}$

$\frac{2}{3y} = \frac{2}{3(-1)} = -\frac{2}{3}$ $\therefore \frac{2}{3x} < \frac{2}{3y}$ ✗

II. $6 - 2x = 6 - 2(-2) = 10$

$6 - 2y = 6 - 2(-1) = 8$ $\therefore 6 - 2x > 6 - 2y$ ✓

III. $-xy = -(-2)(-1) = -2$

$-y^2 = -(-1)^2 = -1$ $\therefore -xy < -y^2$ ✓

14. C

The 1st pattern: number of dots 第 1 個圖案的點子數量 $= 1^2 + 0 = 1$

The 2nd pattern: number of dots 第 2 個圖案的點子數量 $= 2^2 + 1 = 5$

The 3rd pattern: number of dots 第 3 個圖案的點子數量 $= 3^2 + 2 = 11$

The 4th pattern: number of dots 第 4 個圖案的點子數量 $= 4^2 + 3 = 19$

⋮

The 7th pattern: number of dots 第 7 個圖案的點子數量 $= 7^2 + 6 = 55$

15. A

$$\frac{\sin(90° - \theta)\tan(360° - \theta)}{\cos(180° + \theta)} = \frac{\cos\theta(-\tan\theta)}{-\cos\theta} = \tan\theta$$

16. D

$\angle ACB = \angle ADC = \theta$

$AC = CD\sin\theta = x\sin\theta$

$BC = AC\cos\theta = x\sin\theta\cos\theta$

17. D

The exterior angle of the polygon 該多邊形的一外角 $=\dfrac{360°}{n}$

The interior angle of the polygon 該多邊形的一內角 $=\dfrac{360°}{n}+90°$

$$\frac{360°}{n}+\frac{360°}{n}+90°=180°$$
$$n=8$$

I. The value of n is 8. ✓

n 的值為 8。

II. The interior angle of the polygon = $\dfrac{360°}{8}+90°=135°$ ✓

該多邊形的一內角為 135°。

III. The number of axes of reflectional symmetry of the polygon is 8. ✓

該多邊形的反射對稱軸的數目為 8。

18. C

Volume 體積

$=(8\times12-6\times8)\times10$

$=480\text{ cm}^3$

19. D

Let r_A, r_B and r_C be the radius of *A*, *B* and *C* respectively.

設 r_A、r_B 及 r_C 分別為 *A*、*B* 及 *C* 的半徑。

Let A_A, A_B and A_C be the curved surface area of *A*, *B* and *C* respectively.

設 A_A、A_B 及 A_C 分別為 *A*、*B* 及 *C* 的曲面面積。

Let V_A, V_B and V_C be the volume of *A*, *B* and *C* respectively.

設 V_A、V_B 及 V_C 分別為 *A*、*B* 及 *C* 的體積。

$$\left(\frac{r_A}{r_B}\right)^3 = \frac{V_A}{V_B} \qquad \left(\frac{r_B}{r_C}\right)^2 = \frac{A_B}{A_C}$$

$$\left(\frac{r_A}{r_B}\right)^3 = \frac{64}{27} \quad , \quad \left(\frac{r_B}{r_C}\right)^2 = \frac{16}{25}$$

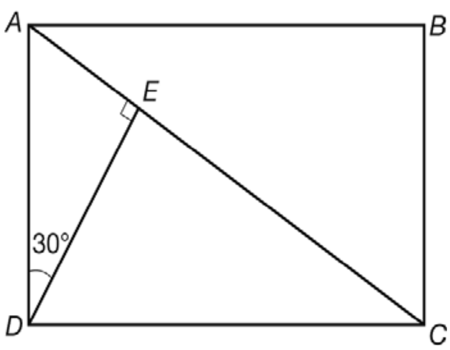
$$\frac{r_A}{r_B} = \frac{4}{3} \qquad \frac{r_B}{r_C} = \frac{4}{5}$$

$\therefore \quad r_A : r_B : r_C = 16 : 12 : 15$

$\therefore \quad r_A : r_C = 16 : 15$

20. D

Let $AD = x$ cm, then $DE = x\cos 30^{\circ}$.



$$\frac{1}{2}(x)(x\cos 30^{\circ})\sin 30^{\circ} = 3$$

$$\frac{\sqrt{3}}{8}x^2 = 3$$

$$x^2 = 8\sqrt{3}$$

$\angle CAD = 60^{\circ}$

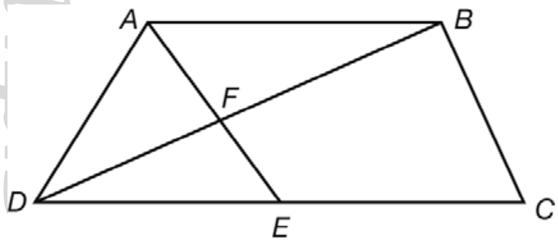
$CD = AD\tan 60^{\circ} = \sqrt{3}x$ cm

$\therefore \quad \text{Area} = AD \times CD = x \times \sqrt{3}x = \sqrt{3}x^2 = \sqrt{3}(8\sqrt{3}) = 24\text{ cm}^2$

21. C

∵ $\Delta DEF \sim \Delta BAF$ (AAA)

∴ $\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta BAF} = \left(\dfrac{DE}{BA}\right)^2$



$\dfrac{25}{\text{Area of } \Delta BAF} = \left(\dfrac{2.5}{3}\right)^2$

$\text{Area of } \Delta BAF = 36 \text{ cm}^2$

∵ ΔDEF and ΔDFA have the same height.

ΔDEF 及ΔDFA 有相同的高。

∴ $\dfrac{\text{Area of } \Delta DEF}{\text{Area of } \Delta DFA} = \dfrac{EF}{FA}$

$\dfrac{25}{\text{Area of } \Delta DFA} = \dfrac{2.5}{3}$

$\text{Area of } \Delta DFA = 30 \text{ cm}^2$

∵ ΔDAE and ΔDBE have the same height and base.

ΔDAE 及ΔDBE 有相同的高和底。

∴ Area of ΔDBE = Area of ΔDAE

= 25 + 30

= 55 cm^2

∵ ΔCBE and ΔDBE have the same height and base.

ΔCBE 及ΔDBE 有相同的高和底。

∴ Area of ΔCBE = Area of ΔDBE

= 55 cm^2

∴ Area of $ABCD$ = Area of ΔCBE + Area of ΔDBE + Area of ΔDFA + Area of ΔBAF

= 55 + 55 + 30 + 36

= 176 cm^2

22. D

The new rectangular coordinates of $P = (-2, -2\sqrt{3})$

P 的新直角坐標$= (-2, -2\sqrt{3})$

Let the polar coordinates of new $P = (r,\ \theta)$

設 P 的新極坐標$= (r,\ \theta)$

$$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = 4$$

$$\tan\theta = \frac{-2\sqrt{3}}{-2}$$

$\theta = 60°$(rejected) or $\theta = 240°$

$\therefore \quad P = (4,\ 240^{o})$

23. B

$$-\frac{1}{-3} \times -\frac{k}{-3} = -1$$

$$k = -9$$

24. D

Slope 斜率$= -\dfrac{1}{a}$

$\because$ Slope is positive 斜率為正數。

$\therefore \quad -\dfrac{1}{a} > 0$

$a < 0$

x-intercept $= -\dfrac{b}{1} = -b$

$\because$ x-intercept is positive.

x-截距為正數。

$\therefore \quad -b > 0$

$b < 0$

25. C

$$\left(\frac{1}{4000}\right)^2 = \frac{25}{\text{the actual area實際面積}}$$

the actual area實際面積 $= 400000000 \text{ cm}^2 = 40000 \text{ m}^2$

26. A

Let $x = 3k$, $y = 2k$, $z = 5k$

$(3x - y - z) : (2x + 3y - 2z)$

$= [3(3k) - (2k) - (5k)] : [2(3k) + 3(2k) - 2(5k)]$

$= 2k : 2k$

$= 1 : 1$

27. B

$$z = \frac{k\sqrt{y}}{x^2}$$

$$y = \left(\frac{x^2 z}{k}\right)^2 = \frac{x^4 z^2}{k^2}$$

New $y' = \dfrac{(0.8x)^4(1.875z)^2}{k^2} = \dfrac{1.44x^4z^2}{k^2} = 1.44y$

Percentage change 百分變化 $= \dfrac{1.44y - y}{y} \times 100\% = 44\%$

28. C

The least possible value of n

n 的最少可能值

$= \dfrac{(50-0.5)\times 1000}{(60+0.5)} \approx 818.182 \approx 818$

29. C

Join OB,

$\angle AOB = 2\angle ADB = 2x$ ($\angle$ at centre = 2$\angle$ at Θ^{ce})(圓心角兩倍於圓周角)

$\angle OBC = \angle OCB = y$ (base$\angle$s, isos.Δ)(等腰Δ底角)

$\angle BOC = 180° - 2y$ ($\angle$ sum of Δ)(Δ內角和)

$\angle AOC = \angle AOB + \angle BOC = 2x + 180° - 2y = 180° + 2x - 2y$

30. A

$\because$ the mode is 7

眾數為 7。

$\therefore$ $x = 7$

$\because$ the median is 6

中位數為 6。

$\therefore$ $y = 6$

Mean 平均值$= \dfrac{2+3+3+6+7}{5} = 4.2$

31. A

$$\frac{1}{m-2} - \frac{1}{2+m} = \frac{1}{m-2} - \frac{1}{m+2} = \frac{m+2-(m-2)}{m^2-4} = \frac{4}{m^2-4}$$

32. C

$(2i)^{100}+(-2i)^{100}=2^{100}i^{100}+(-2)^{100}i^{100}=2^{100}(1)+(2)^{100}(1)=2\times(2)^{100}=2^{101}$

33. A

$$\alpha^2-3\alpha+1=0$$
$$2\alpha^2-6\alpha+2=0$$
$$2\alpha^2-6\alpha-1+2=-1$$
$$2\alpha^2-6\alpha-1=-1-2$$
$$2\alpha^2-6\alpha-1=-3$$

34. C

35. D

Let $a=k\log 3$ and $b=k\log 2$,

$$\begin{aligned}(a+b):(a+3b)&=(k\log 3+k\log 2):(k\log 3+3k\log 2)\\&=[k(\log 3+\log 2)]:[k(\log 3+\log 2^3)]\\&=(k\log 6):(k\log 24)\\&=\log 6:\log 24\end{aligned}$$

36. D

$m^2-2m+1=(m-1)^2$

$m^2-1=(m+1)(m-1)$

$m^3-1=(m-1)(m^2+m+1)$

L.C.M. $=(m-1)^2(m+1)(m^2+m+1)$

37. A

$$\begin{aligned}11001000010001_2&=2^{13}+2^{12}+2^9+2^4+2^0\\&=2^{13}+2^9+2^{12}+2^4+2^0\\&=2^{13}+2^9+4096+16+1\\&=2^{13}+2^9+4113\end{aligned}$$

38. B

Join *CD*,

$\angle BDC = \angle CBG = 40°$ (∠ in alt.segment)(交錯弓形的圓周角)

$AE = DE$ (tangent properties)(切線性質)

$\angle EDA = \angle EAD = \dfrac{180° - 70°}{2} = 55°$

$\angle DCA = \angle EAD = 55°$ (∠ in alt.segment)(交錯弓形的圓周角)

$\angle CHD = 180° - 40° - 55° = 85°$ (∠ sum of Δ)(Δ內角和)

39. B

$$\begin{cases} y = 2\sin^2 x & \text{..........(1)} \\ y = -3\cos x & \text{..........(2)} \end{cases}$$

Put (1) into (2),

$$\begin{aligned} 2\sin^2 x &= -3\cos x \\ 2(1-\cos^2 x) + 3\cos x &= 0 \\ 2 - 2\cos^2 x + 3\cos x &= 0 \\ 2\cos^2 x - 3\cos x - 2 &= 0 \\ (\cos x - 2)(2\cos x + 1) &= 0 \\ \cos x = -\frac{1}{2} \quad &\text{or} \quad \cos = 2 \text{ (rejected)} \\ x = 120° \quad &\text{or} \quad 240° \end{aligned}$$

∴ There are two points of intersections.

有兩個相交點。

40. C

Put (0, 2) into $y = k\cos\left(\frac{1}{2}x + \theta\right) + 2$,

$2 = k\cos\left(\frac{1}{2}(0) + \theta\right) + 2$

$2 = k\cos\theta + 2$

$0 = \cos\theta$

$\theta = 90°$ or $\theta = 270° = -90°$

Put (180°, 5) and $\theta = 90°$ into $y = k\cos\left(\frac{1}{2}x + \theta\right) + 2$,

$5 = k\cos\left(\frac{1}{2}(180°) + 90°\right) + 2$

$5 = k\cos(180°) + 2$

$5 = -k + 2$

$k = -3$

Put (180°, 5) and $\theta = -90°$ into $y = k\cos\left(\frac{1}{2}x + \theta\right) + 2$,

$5 = k\cos\left(\frac{1}{2}(180°) - 90°\right) + 2$

$5 = k\cos(0°) + 2$

$k = 3$

41. B

Note that ΔOAB is a right-angled triangle.

留意ΔOAB為一直角三角形。

∴ The circumcentre of ΔOAB is the mid-point of AB, i.e., (3, 4), let's denote the point by M.

ΔOAB 的外心為 AB 的中點，即(3, 4)，將該點標記為 M。

The orthocentre of ΔOAB is O.

ΔOAB 的垂心為 O。

Note that OM is a median of ΔOAB.

留意 OM 為ΔOAB 的中線。

∴ The centroid, circumcentre and orthocentre of ΔOAB lie on the same straight line, OM.

ΔOAB 的形心、外心及垂心位於同一直線 OM 上。

42. C

The number of possible passwords

可組成的密碼數目

$= 26\times(C_1^{25}\times C_2^{10}\times 3!+P_3^{10})$

$= 194220$

43. C

The required probability

所求概率

$=1-\dfrac{2}{10}-\dfrac{8}{10}\times\dfrac{2}{9}$

$=\dfrac{28}{45}$

44. C

$$\begin{cases} x-y+k=0 \text{........(1)} \\ x^2+y^2=2 \text{ ....(2)} \end{cases}$$

From (1), $y=x+k$........(3)

Put (3) into (2),

$$x^2+(x+k)^2=2$$
$$2x^2+2kx+k^2-2=0$$
$$\Delta=0$$
$$(2k)^2-4(2)(k^2-2)=0$$
$$4k^2-8k^2+16=0$$
$$k^2=4$$
$$k=2 \quad \text{or} \quad k=-2 \text{ (rejected)}$$

Let (a, b) be a point on L such that it is at the shortest distance from C_2.
設(a, b)為 L 上的點使它與 C_2 為最短距離。

$$b=a+2$$
$$a-b=-2 \text{........(1)}$$

Slope of L = 1
Centre 圓心= (4, –2), Radius 半徑= 1

$$\frac{b-(-2)}{a-4}\times 1=-1$$
$$b+2=-a+4$$
$$a+b=2 \text{........(2)}$$

By solving (1) and (2), we have a = 0, b = 2.
解(1)及(2)，得出 $a=0$，$b=2$。

∴ Shortest distance 最短距離

$$=\sqrt{(4-0)^2+(-2-2)^2}-1=\sqrt{32}-1=4\sqrt{2}-1$$

45. C

Centre 圓心= (0, 6)

Radius 半徑= $\sqrt{0^2+6^2-12}=\sqrt{24}=2\sqrt{6}$

Distance between centre and the origin 圓心與原點的距離

= 6

$$\therefore \quad \tan\theta=\frac{2\sqrt{6}}{\sqrt{6^2-(2\sqrt{6})^2}}=\frac{2\sqrt{6}}{\sqrt{12}}=\frac{2\sqrt{6}}{2\sqrt{3}}=\frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}}=\sqrt{2}$$